Abstracts
5th International Workshop on Nonlinear Analysis and its Applications

Sridev Suman Uttarakhand University
Rishikesh (Uttrakhand), INDIA

December 14-16, 2023

Preface

This abstract booklet includes the abstracts of the papers announced for the 5th International Workshop on Nonlinear Analysis and its Applications *IWNAA2023*, held at Sridev Suman Uttarakhand University, Rishikesh (Uttrakhand), India during December 14-16, 2023.

The series of International Workshop on Nonlinear Analysis and its Application, IWNAA 2018, was initiated by Juan MARTINEZ-MORENO and Antonio Francisco ROLDÁN-LÓPEZ-DE-HIERRO on July 7-9, 2018, in Granada University, Spain. The second of this series was done on September 12-14, 2019 in Bangsaen, Chonburi, Thailand under the name of International Workshop on Applied Nonlinear Analysis IWANA 2019. Due to Covid 19 pandemic, the edition 2020 was cancelled by the organizing committe. In 2021, we organized the third one virtually and we decided to continue under the name of International Workshop on Nonlinear Analysis and its Application (IWNAA). The third edition, IWNAA 2021, was organized by Marija CVETKOVIC from 13th to 16th of October at the Faculty of Science and Mathematics, University of Niš. The fourth meeting of the series, IWNAA 2022, was organized by Mahpeyker ÖZTÜRK, from 12th to 15th of October at Sakarya University of Applied Sciences and Sakarya University.

The purpose of the workshop is to bring together leading experts and researchers in nonlinear analysis, in particular, fixed point theory, and to assess new developments, ideas, and methods in this important and dynamic field. A special emphasis will be put on applications in related areas, as well as other sciences, such as the natural sciences, medicine, economics, and engineering.

The conference brings together more than XXX participants from XXX countries (Algeria, Australia, Bulgaria, Canada, China, Germany, India, Iran, Japan, Mexico, Nigeria, Oman, Pakistan, Poland, Romania, Russia, Saudi Arabia, Serbia, South Africa, South Korea, Spain, Taiwan, Thailand, Tunisia, Turkey, United Arab Emirates, USA), out of which XXXX are contributing to the meeting with oral, including four Plenary talks. Some fields covered in these presentations include Fixed Point Theory, Dynamical Systems, Fractional Differential Equations, Dynamic Equations, Numerical Analysis, Modeling, and PDEs with applications.

This is the fifth meeting of the series. IWNAA2023 will be organised by Anita TOMAR, virtually, from 14th to 16th of December at Pt. L. M. S. Campus, Sridev Suman Uttarakhand University, Rishikesh (Uttrakhand) India.

We wish everyone a fruitful conference.

Anita TOMAR Chair

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Contents

Main Speakers

A nonlinear theoretic application of Li-Yorke theorem to Collatz conjecture

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The Collatz conjecture is a conjecture named after Prof. Collatz, who first proposed it in 1937, and so far, there exist several suggestive approaches to this conjecture, which are based on various mathematical research areas such as number theory, probability theory and computation theory, This famous conjecture can be represented as the following:

Let $\mathbb{N}$ be the set of all positive integers. Then Collatz mapping whose domain and image are $\mathbb{N}$ can be defined as the following:

$$C(n) = \begin{cases} 3n+1, & n \text{ is odd}, \\ \frac{n}{2}, & n \text{ is even}. \end{cases}$$

Then, for any positive integer n, we can find a certain positive integer $k(n)$ satisfying the following:

$$C^{k(n)}(n) = 1,$$

where $c^{k(n)}(\cdot)$ means the $k(n)$-time nested superposition of $C(\cdot)$.

In the theory of dynamical systems, Li-Yorke theorem can be represented as the following:

If a real-valued continuous function of one real variable has a periodic point whose prime period is three, then, for any positive integer $n \geq 3$, the function has a periodic point whose prime period is equal to n.

In this talk, we introduce an example introducing how to apply Li-Yorke theorem to Collatz conjecture.

Keywords: Collatz' conjecture, Li-Yorke theorem.

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Chasing the equivalence

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The question of novelty and applicability is often taken into consideration when a new theoretical result is presented. In the era of generalizations and numerous types of contractions and abstract spaces appearing on an almost everyday basis, we will discuss their mutual relation. Are we making real progress or just running in circles?

Birkhoff-Kellogg type results in cones with applications

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We present some classical and modern results of Birkhoff-Kellogg type and their application to the solvability of parameter-dependent problems in differential equations.

Is there a recession in the metric fixed point theory?

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The main aim of this talk is to indicate the dilemmas of the metric fixed point theory. First, we want to underline that almost all real-world problems are in the context of fixed point theorems. This observation reflects the fact that how metric fixed point theory can be helpful in a vast number of distinct disciplines. In other words, the metric fixed point theory has extensive application potential. On the other hand, several results or their equivalent forms were re-published due to ambitious authors' great interest in the metric fixed point theory. A substantial part of the new results suggested overlap with the current results. This talk aims to underline the difference between the novel and overlap results.

[1] E. Karapinar, M. Cvetkovic, Remarks on some generalizations of theta-contraction, University Politehnica of Bucharest Scientific Bulletin-Series A-Applied Mathematics and Physics , Volume 85 Issue2 Page31-42, 2023

[2] E. Karapinar, Recent Advances on Metric Fixed Point Theory: A Review, Applied and Computational Mathematics an International Journal, Vol 22. issue 1, 2023, pages: 3-30, www.acmij.az/view.php?lang=azmenu=8

Convex minimization problems on a complete geodesic space with curvature bounded above and approximation technique of their solutions

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The convex minimization problem is a central topic in convex analysis, and it has been considered mainly in functional spaces such as Hilbert spaces and Banach spaces. This study has recently been generalized to the setting of Hadamard spaces and other geodesic spaces, and many researchers have proposed various approximation techniques to solve this problem. In this work, we focus on approximating schemes using resolvent operators defined for convex functions and propose a new type of iterative scheme.

On some vector optimization problems

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Some vector optimization problems are studied from the point of view of their efficiency and proper efficiency. An adequate generalized convexity jointly with a parametric approach are used to formulate results on the Pareto optimality for primal and dual minimizing problems.

Differential equations with distributed delay: existence and asymptotic stability of solutions

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We discuss the existence and asymptotic stability of solutions of the partial differential equation arising from population dynamics models

$$\frac{\partial u}{\partial t}(t,x) = -b(t,x)u(t,x) + g\left(t, u(t,x), \delta_T(t,x,u)\right), t \geq t_0, x \in [0,1]. \qquad (1)$$

The evolution of the process is conditioned by the effects of a distributed delay described by δ_T. The positive number T provides the width of the action of the delay: the larger T, the more the system's memory is extended to past events affecting its present state. Inasmuch as the process is set on the whole half-line, there are no a priori bounds to the number T. In our study we consider $\delta_T(t,x,u) = \int_{-T}^{0} u(t+\theta,x)d\theta$ or $\delta_T(t,x,u) = \int_{t_0}^{t} \frac{e^{-(t-s)/T}}{T} u(s,x)ds$. The first expression of δ_T is used when the process at each instant t has memory of the evolution of the state of the system up to that moment for a past of amplitude T. This indicates a functional dependence of the law of development of the phenomenon on the past trajectory. By means of appropriate positions, equation (1) can therefore be reformulated as a functional semilinear differential equation in abstract spaces of the form

$$y'(t) = A(t)y(t) + f\left(t, y(t), y_t\right), t \geq t_0, \qquad (2)$$

where y_t stands for the function $y_t(\theta) = y(t+\theta), \theta \in [-\tau, 0]$. The second expression of δ_T is instead used when events are less and less relevant to the present state of the system as they go further back in time. This phenomenon is described by the integral term whose kernel is the exponential probability distribution $\mathcal{K}(\tau) = me^{-m\tau}$ with decay parameter $m = \frac{1}{T}$, being T the historical average waiting time. In this case we review equation (1) as a semilinear integro-differential equation of the type

$$y'(t) = A(t)y(t) + f\left(t, y(t), \int_{t_0}^{t} k(t,s)y(s)ds\right), t \geq t_0, \qquad (3)$$

where k is a continuous real function. We show how, by combining iterative methods and fixed point theorems for condensing maps, one can obtain the existence and asymptotic stability of the solutions of the differential equations (2) and (3), from which then the expected results for the model equation (1).

On Some Open Problems related to Fixed Point Theorems of Nonexpansive Mappings

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If K is a nonempty closed bounded convex subset of a Hilbert space X and $T : K \to K$ is a nonexpansive map then we know that there exists a point x in K such that $Tx = x$. On the other hand if K is a nonempty closed bounded convex subset K of X with respect to an equivalent norm on X and $T : K \to K$ is a nonexpansive map with respect to this equivalent norm then it is not known whether there exists a point x in K such that $Tx = x$. In this talk we intend to discuss such open problems related to fixed point theorems of nonexpansive mappings.

Some recent results on double-phase problems

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We consider a special form of double-phase differential operator under logarithmic perturbation, and discuss its features in the setting of Musielak-Orlicz spaces. We also establish the existence of weak solutions to certain double-phase Dirichlet problems with general nonlinearities.

Invited Speakers

On time-scale compartmental epidemic models

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Mathematical modeling of infectious diseases has been used to study the spread mechanism of such diseases with their risk factors and to find strategies to control an epidemic. Kermack and McKendrick formulated the SIR (Susceptible-Infected-Recovered) compartmental model as a pioneering mathematical epidemiology model in 1927. Several extensions of their basic model have been studied by many epidemiologists and mathematicians through various continuous and discrete compartmental models. In this talk we first summarize some epidemic dynamic models aiming to unify the continuous and discrete models. Then on time scales we propose a SIR model and its modified SEIR (Susceptible-Exposed-Infected-Recovered) model where there is a latent period of the disease where the person has been infected but is not yet infectious. We investigate the solvability and some properties of the solutions of the systems of dynamic equations by using the fixed point theory.

Some recent results on fixed point theory in connection with initial and boundary value problems

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One of the most important applications of the fixed point theory is the investigation of existence and uniqueness of solutions of various types of equations: linear, nonlinear, algebraic, ordinary or partial differential equations, functional, integral equations etc. Together with the recent developments in fixed point theory including the numerous extensions of the metric space and the contraction mappings, the problem of existence and uniqueness of solutions for these various types of equations has been reconsidered and, in many cases the conditions imposed on the equations and/or the accompanying conditions have been refined.

The purpose of this talk is to present some recent studies related with the applications of fixed point theory to the existence-uniqueness problems. These problems include boundary value problems associated with fractional differential equations, dynamic, integral, integro-dynamic and fractional dynamic equations on time scales and finally some matrix algebraic equations. The existence-uniqueness results are illustrated with examples and numerical computations.

Some results on spectral theory for suprema preserving operators on max-cones in normed vector lattices

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Suprema preserving nonlinear operators are interesting from the applicative and theorethical point of view. Although such operators may usually be sensibly defined on the whole normed vector or Banach lattice, they are often well behaved (suprema preserving, positively homogeneous, Lipschitz, ...) on some smaller cone. In the talk some elements of the spectral theory for such operators will be presented. For example, the eigenproblem of such operators naturally arises in the asymptotic study of periodic solutions of a class of differential-delay equations. Some analogous results for cone linear operators on normal cones in normed spaces will be pointed out.

The talk is mainly based on three joint papers with Müller and Peperko [1-3].

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Pata type Best Proximity Point Results

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The aim of this talk, is to study of best proximity point and optimal coincidence point results of some α Pata-proximal admissible contraction of type-I and type-II in the framework of complete metric space. Some examples are presented to support the results obtained herein. Further, we will unify, extend and generalize various existing results in literature. We will also try to elaborate the main concern or issues related to fixed point theory in recent time.

Keywords: Metric space and α-Pata-proximal admissible contraction of type-I and type-II and Generalized α-Pata-proximal contraction and Optimal coincidence best proximity point.

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Solvability of the boundary value problem for the q-difference equation

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Solvability conditions of the boundary value problem for the q-difference equation are discussed by Banach contraction mapping theorem on b-metric spaces. The problem is studied by considering a fixed point problem for an integral operator after it is transformed into an integral equation. Moreover, an example is introduced to provide the theoretical results.

On Topological Classification of Fractal Square Dendrites

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A fractal square of order n with a *digit set* $D \subset \{0, ..., n-1\}^2$ is the unique compact set $K \subset [0,1]^2$, which satisfies the equation $K = \dfrac{K+D}{n}$. If the set K is a dendrite, we call it a Fractal Square Dendrite, or FSD. If K is an FSD, then it possesses one-point intersection property and its intersection graph is a tree.

Moreover, the self-similar boundary ∂K of an FSD K is finite. We show it may be of 5 possible types and may consist of 3,4,5 or 6 points. The minimal topological subtree $\gamma \subset K$, containing ∂K, is called *the main tree* of K. We prove that the main trees of FSD K belong to 7 possible classes. Taking into account the placement and orders of the points of ∂K with respect to the main tree γ, we prove that there are 15 possible types of main trees for non-degenerate fractal square dendrites.

Contributed Talks

Max-product operators

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In this paper, we prove the convergence in Lebesgue spaces with variable exponent $L^{q(\cdot)}(\mathcal{R})$ with $1 \leq q^- \leq q(x) \leq q^+ < +\infty, \forall x \in \mathcal{R}$, for the multivariate max-product neural network operators of the Kantorovich type activated by sigmoidal functions. Also, the bounded and uniform convergence has been proved for functions belonging to suitable spaces. The max-product operators find applications in problems related to probability and fuzzy theory.

Extended m_v^b-metric space and solution of rocket's ascending motion

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We introduce extended m_v^b-metric space and extended m_v-metric space with a focus on their topological notions including relations between m_v-metric, extended $b_v(s)$-metric and extended m_v-metric. By challenging the conventional assumption of zero self-distance with $v+2$ points and a coefficient greater than 1, we investigate novel postulates to establish the fixed point theorems in these domains and pave the way for more accurate mathematical models applicable to real-world scenarios. Our findings are supported by illustrative examples. Additionally, using our result we explore the problem that indicates ascending motion of a rocket. Consequently, our research contributes not only to a deeper comprehension of mathematical concepts but also to practical utility across various scientific domains.

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A Viscosity Approach to Tackle a Φ-contraction Mapping for Hierarchical Variational Inequality Problem

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In this paper, we propose the viscosity method for solving variational inequality problems defined over a set of fixed points of a nonexpansive mapping and involving a φ-contraction mapping and another nonexpansive mapping in the setting of Hadamard manifolds. Several special cases of such a variational inequality problem are also considered. The convergence analysis of the proposed method is studied. We illustrate proposed algorithm and convergence result by a numerical example. The algorithms and convergence results of this paper extend and improve several known algorithms and results from linear structure to Hadamard manifolds.

On a new nonlinear convex structure

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In this work we start from near vector spaces, which we endow with some additional properties that allow convex analysis. The seminormed structure used here will also be improved, by adding properties such as the null condition and null equality, thus resulting in a new type of space, which is still weaker than the conventional Banach structures: pre-convex regular near Banach space. On the newly defined structure, we introduce the concept of uniform convexity and analyze several resulting properties. The major outcomes prove a remarkable resemblance to the classical properties resulting from uniform convexity on hyperbolic metric spaces or modular function spaces, including the famous Browder-Gohde fixed point theorem.

Generalized contractive inequalities

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In this paper I consider generalized contractive inequalities which extend some inequalities which are already known. I prove that mappings which satisfy them have fixed points which are unique. The setting used is that of strong b-metric spaces. Another direction of extensions refers to the underlying space used to prove fixed point results. Generalization had in view dropping one or more conditions from the metric definition, or enlarge the triangle inequality.

On the zero existence problem for a multivalued functional connected with a family of search functionals.

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The talk is based on our recent joint work with A. Kurbanov. We present a continuation of previous author's works (see [1,2] at alias) on the zero existence problem for multivalued (α, β)-search functionals on a metric space. More exactly, we investigate the zero existence problem for a functional connected with a parametric family of search functionals on an open subset of a metric space. In our joint work with Yu. Zakharyan [3], we had obtained a theorem on the zero existence preservation, by changing the parameter, for a parametric family of multivalued (α, β)-search functionals on an open subset of a metric space. As one of the corollaries of that result, we had obtained an essential generalization of the fixed point existence continuation theorem by M. Frigon [4], for a contraction parametric family of multivalued mappings.

Definition [3]. Let (X, d) be a metric space, $Y \subseteq X$. A multivalued functional $\Phi : Y \rightrightarrows \mathbb{R}_+$ is called *search on* Y (see [3]), if there are $\alpha, \beta, 0 \leq \beta < \alpha$, such that for $x \in Y$, $c \in \Phi(x)$, and $r > 0$, with $\overline{B(x; r)} \subset Y$, $c \leq (\alpha - \beta)r$, there is a point $x' \in \overline{B(x; c/\alpha)}$ and a value $c' \in \Phi(x')$ such that $c' \leq (\beta/\alpha)c$.

Now, we consider a multivalued functional φ which is not (α, β)-*search on* Y, but is connected with a family $\{\Phi_t\}_{0 \leq t < 1}$ of search multivalued functionals by means of the θ-continuity property. That is, there is an increasing continuous function $\theta : [0; 1] \to \mathbb{R}_+$ such that for any $(x, t) \in U \times [0; 1)$, with $0 \in \Phi_t(x)$, there exists $c \in \varphi(x)$ satisfying the condition $c \leq |\theta(1) - \theta(t)|$. The main result of the talk is a theorem containing some versions of sufficient conditions, for the solving this problem. In addition, the metric counterparts are proved in a uniformly convex metric space of the asymptotic center existence theorem [4], and of the fixed point theorem for a non-expansive multivalued mapping [5]. The comparison of the latter results with the main theorem is carried out as well.

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Some results to algebras.

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Let A and B be Banach algebras with $\sigma(B) \neq \emptyset$. Let $\theta, \phi, \gamma \in \sigma(B)$ and $\mathrm{Der}(A \times_\theta^{\phi,\gamma} B)$ be the set of all linear mappings $d : A \times B \to A \times B$ satisfying $d((a,b) \cdot_\theta (x,y)) = d(a,b) \cdot_\phi (x,y) + (a,b) \cdot_\gamma d(x,y)$ for all $a, x \in A$ and $b, y \in B$. In this talk, we characterize elements of $\mathrm{Der}(A \times_\theta^{\phi,\gamma} B)$ in the case where A has a right identity. We then investigate the concept of centralizing for elements of $\mathrm{Der}(A \times_\theta^{\phi,\gamma} B)$ and determine dependent elements of $\mathrm{Der}(A \times_\theta^{\phi,\gamma} B)$. We also apply some results to group algebras.

The noncompact Schauder fixed point theorem in random normed modules and its applications

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Motivated by the randomized version of the classical Bolzano–Weierstrass theorem, in this paper we first introduce the notion of a random sequentially compact set in a random normed module and systematically develop the related theory. Based on these developments, we prove the corresponding Schauder fixed point theorem: let E be a random normed module and G a random sequentially compact L^0-convex set of E, then every σ-stable continuous mapping from G to G has a fixed point, which unifies all the previous random generalizations of Schauder fixed point theorem. As one of applications of the theorem, we prove the existence of Nash equilibrium points in the context of conditional information. It should be pointed out that the main difficulty of our whole paper lies in overcoming noncompactness since a random sequentially compact set is very often noncompact.

Fractional differential equations with generalized proportional Riemann-Liouville fractional derivatives and impulses:Stability

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One of the main properties studied in the qualitative theory of differential equations is the stability of solutions. There are various types of differential equations with different initial or boundary value conditions. Recently, several types of fractional derivatives are defined, studied and applied to model the dynamics or real processes and phenomena. Stability of fractional order systems is quite recent. There are several approaches in the literature to study stability, one of which is the Lyapunov approach. However the Lyapunov approach to fractional differential equations causes many difficulties. In this paper we study differential equations with generalized proportional fractional derivative of Riemann0Liouville type. Also, impulses are involved in the equation. The applied type of fractional derivative requires a new definitions of stability excluding the initial point as well as the impulsive times. Comparison results are presented and sufficient conditions for stability are given. Examples are presented to illustrate the theory.

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New Generalization of Interpolative Contractions by means of Fixed Point

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This talk is organized for young people to aware about Fixed Point Theory. In this conference present new idea of interpolative contractions and build up some new results for such contraction in the context of F-metric space. We derive some consequences and examples to support out interpolative contractions in F-metric spaces. The inspiration of this study is to observe the solution of Fredholm integral and fractional-order differential equation with one of the boundary conditions using fixed-point technique in F-metric space.

Keywords: Fixed point, Interpolative Contractions, F-metric spaces, fractional differential equation.

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Birkhoff-Kellogg type results in cones with applications

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We present some classical and modern results of Birkhoff-Kellogg type and their application to the solvability of parameter-dependent problems in differential equations.

Solution to Elastic Beam Differential Equation via Coupled Fixed Point Results in FMS

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This communication deals with the establishment of the unique solution of the differential equation of an elastic beam problem in physical sciences via coupled fixed point results in GV-FMS with H-type t-norm. First, we establish the existence and uniqueness of coupled fixed points for the mixed monotone operators under a non-linear contraction in the setting of partially ordered GV-FMS endowed with H-type t-norm. The computational and graphical analysis of the applied example illustrates the experimental verification of the coupled fixed point result. Then, the obtained results are applied to attain the unique solution of the said elastic beam differential equation.

Keywords: Partially ordered set, Hadzic type t-norm, Coupled common fixed point, GV-FMS.

Geometry and Continuity of Fixed Point in Partial Metric Space

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To explore unique and non-unique fixed points along with their geometric implications, we introduce k-continuity and weaker orbital continuity within the innovative context of Partial Metric Spaces driven by practical requirements in computer science. This research addresses continuity issues at fixed points, departing from conventional metric spaces and accommodating non-zero self-distances, thus highlighting the synergy between mathematics and computer science.

Keywords: Partial metric space, Fixed point, k-continuity, weaker orbital continuity.

C_0-semigroups and Local spectral theory

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Let $(T(t))_{t\geq 0}$ be a C_0-semigroup of operators on a Banach space X. In this paper, we show that if there exists $t_0 > 0$ such that $T(t_0)$ has the SVEP then A has the SVEP and if $\sigma_p(A)$ has empty interior, then $T(t)$ has the SVEP for all $t \geq 0$. Also, some local spectral properties for C_0 semigroups and theirs generators and some stabilities results are also established.

Banach Contraction Principle and Beyond: Fixed Point Theorems in Rectangular Modular b-Metric Spaces

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This paper delves into the exploration of rectangular modular b-metric spaces. We commence by establishing fundamental properties of this space and subsequently prove Banach's Fixed Point theorem. Next, we present several fixed-point approaches within this context. Finally, we discuss the practical applications of this space, highlighting that our findings encompass broader scenarios compared to conventional rectangular b-metric spaces or standard modular b-metric spaces.

Common Fixed point Theorem for Hardy-Rogers-type Interpolative Mappings in b-Metric Space with Some Applications

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The novelty of this paper is to describe a common fixed point theorem for Hardy-Rogers-type interpolative mappings in b-metric space with some applications. The results proved in this manuscript are verified with suitable example. Besides this, we demonstrate our main results with the application of Volterra-Fredholm-type integral equations.

References: Common fixed point, interpolative mapping, b-metric space, Volterra-Fredholm type integral equations.

Fuzzy Measure of Non-compactness with Applications in Fractional Anti-periodic Boundary Value Problems Involving Nonsingular Kernel

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In this talk we present the notion of fuzzy measure of noncompactness. We define the fuzzy condensing and fuzzy k-set contractions using fuzzy measure of noncompactness. An extension of Stanislaw Szulfa's fixed point theorem for a self-operator on a closed bounded and convex subset of a Banach space has been proved. Using the defined multivalued k-set contractions, generalizations of Kakutani-Fan and Krasnoselskii type theorems have been proved. For applications the existence result for solutions of a fractional Caputo-Fabrizio anti-periodic boundary value problem has been given with proofs. We give some examples to validate our results.

An iterative numerical method for Fredholm integral equations of Hammerstein type with modified argument

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The paper presents an iterative numerical method for Fredholm-Hammerstein integral equations with modified argument. The numerical scheme consists of two parts: a fixed point result with successive iteration and a suitable Newton-Cotes quadrature formula. We prove the convergence of the method, give error estimates and show its applicability on several numerical examples.

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A Nyström-based iterative technique for solving nonlinear Fredholm integral equations of type II

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The current study provides an iterative numerical system to discover the approximate solution of the nonlinear Fredholm integral equations of the second kind. This scheme is built upon the quasilinearization technique and Nyström method. The nonlinear Fredholm integral equation is first converted to a series of linear Fredholm integral equations using the quasilinearization approach. The unique solution of the initial issue is quadratically approached by the exact solutions of this series of linear integral equations, provided certain conditions are met. Next, the Nyström method is used in each iteration to approximate the linear Fredholm integral equation using the linear barycentric rational quadrature. A detailed investigation is conducted into the method's convergence analysis. Theoretical conclusions are validated and performance is demonstrated by numerical examples and compared with other current approaches.

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On Approximation Theory

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Approximation theory has much wider applications in applied and computational mathematics, engineering and allied fields of science. Out of different types of approximation theory such as approximation of functions using polynomials, trigonometric polynomials, spline, rational functions etc. the present study is dealing with approximation by positive linear operators. Approximation theory involving positive linear operator mainly deals with convergence behavior of sequence of positive linear operator. Many new operators were developed by several researchers after the development of very important convergence theorem of Korovkin. In 1953, P.P. Korovkin presented a simple criterion to determine the convergence of a given sequence of positive linear operators to the desired function.

Modeling and numerical simulation of the penalty method for unilateral contact problem

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In this work, we consider the penalty method applied to contact problem in thermo-electro-visco-elasticity with Signorini's condition and Tresca's friction law. Mathematical properties, such as the existence of a solution to the penalty problem and its convergence to the solution of the original problem, are reported. Then, we present some numerical results in the study of a two-dimensional test problem and we establish its convergence.

Keywords: Thermo-piezo-electric, Tresca's friction, Signorini's condition, penalty method, numerical schemes, finite element approximation, numerical simulation.

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Some of the results obtained in Menger's spaces

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In this talk, we will give a brief overview of some of the results obtained in Menger's spaces recently. One part of these results is related to getting the answer, in the setting of Menger PM spaces, to the question on the existence of contractive mappings which admit discontinuity at the fixed point. In the second part of these results is shown that orbital continuity for a pair of self-mappings is a necessary and sufficient condition for the existence and uniqueness of a common fixed point for these mappings defined on Menger PM-spaces with a nonlinear contractive condition. These results are obtained using some weaker forms of commutativity.

Z-Gornicki mapping and Proximal Z-Gornicki mapping results

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In this work we introduce Z-Gornicki mapping and Proximal Z-Gornicki mapping by using simulation functions [1, 2] in metric spaces and partial metric spaces [3] which generalizes Gornicki mapping [4,5] and Proximal Gornicki mapping. Examples given to illustrate the results.

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Some remarks on L-type mappings

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The notion of L-type mappings was introduced by Enrique Llorens-Fuster and Elena Moreno-Galvez in [2]. In my speech I consider the behaviour of continuous L-type mappings on compact and weakly compact subsets of Banach spaces. These results improve and answer some claims and questions from [1] and [2].

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Hyers−Ulam−Rassias stability theorems via fixed point technique

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We introduce logarithmic and generalized logarithmic set−valued functional equations and inequalities to establish the Hyers−Ulam−Rassias stability theorems via fixed point technique in a real Banach space. We offer substantial illustrations to demonstrate the significance of our investigations concerning the stability of these functional equations.

Boundary Controllability of Sobolev-type Neutral Integrodifferential System With Timevarying Delays

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In this article, sufficient conditions for the boundary controllability of semilinear impulsive Sobolev-type neutral integrodifferential functional evolution systems with timevarying delays in Banach spaces have been established. The results are obtained by using two-parameter family of evolution operators and fixed point theorem. An example is provided to illustrate the theory.

Keywords: Boundary controllability, Neutral integrodifferential equation, impulsive differential equation Semigroup theory, Fixed point theorem

Artificial Neural Networks for Solving Nonlinear Ordinary Differential Equations

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Here, we derive expressions for solving nonlinear ordinary differential equations with the aid of neural networks. Each neuron associated with two operations: aggregation (linear) and activation (nonlinear). Nonlinearity in the architecture of neural networks plays a crucial role. Here, an extensive study of first, second and third ordered nonlinear ordinary differential equations are considered to emphasize the role neural networks. Few celebrated examples are considered for this study to draw the attention of nonlinear operation occurred with each neuron.

On some fixed point results and extension of Karlovitz theorems

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In this work, we solve two fixed point problems associated with the class of (c)-mappings. The first one is devoted to obtain the existence of fixed points for such mappings defined on weak* closed bounded convex subsets of duals of separable Banach spaces when the orthogonality relation $\perp$ is uniformly weak* approximately symmetric. For the second problem, using the idea of R. Smarzewski (on firmly nonexpansive mappings, Proc. Am. Math. Soc 113(3):723-725,1991), we prove the existence of fixed points for such mappings which are defined on a finite union of weakly compact convex subsets of uniformly convex in every direction Banach spaces.

Keywords: fixed point, (c)-mapping, asymptotically regular, approximate fixed point sequence, weak* closed convex subset, weak* convergence, orthogonality, approximately symmetric, weak? approximately symmetric, dual Banach space, uniformly convex Banach space in every direction.

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On fixed points and generalisations of metric space

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Banach contraction principle is one of the most important result in analysis. This result has been extended or generalized in different directions. Generalisation of metric space is one of the important direction in which Banach fixed point theorem is extended. In this talk, we discuss about some important generalisations of metric space namely b-metric space, rectangular metric space, S-metric space, G-metric space and their generalisations. Main discussion is about the weak points of the generalised spaces specially in examining the existence of fixed points.

Keywords: Banach contraction principle, fixed point, b-metric space, S-metric space, G-metric space, rectangular metric space.

Proximal Normal Structure on Banach Spaces

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Eldred, Kirk and Veeramani introduced the notion of proximal normal structure to prove the best proximity point theorems for relatively nonexpansive mappings and showed that uniformly convex Banach spaces has the proximal normal structure. In this paper, a characterization for the weak proximal normal structure is given. Using this characterization, it is proved that every weakly compact convex pair in a Banach space X has proximal normal structure whenever X satisfies: X is ϵ_0-inquadrate in every direction for some $\epsilon_0 \in (0,1)$ or X has the modulus of $k - UC\delta_X^k(1) > 0$, for $k \in \mathbb{N}$ or X has the modulus of k dimensional U-convexity $U_X^k(1) > 0$, for $k \in \mathbb{N}$ or X has the coefficient of noncompact convexity $\epsilon_1(X) < 1$. Moreover, we generalize the notion ϵ_0-inquadrate in every direction to ϵ_0-inquadrate with respect to every k-dimensional subspace and showed that X has the weak proximal normal structure and the weak normal structure if $\epsilon_0 \in (0,1)$. In case of $\epsilon_0 \in [1,2)$, the Banach space X has the weak proximal normal structure with an additional assumption X has the WORTH property.

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Chaos and Fractals in Nonlinear Dynamical Systems

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Almost all scientific and engineering disciplines, chaos and fractals are widely observed. Many physical process can be modelled in terms of mathematical expressions which generally lead to equations. We can visualize the behaviour of solutions effectively through fractals. Chaos exists everywhere in the world since most of problems are nonlinear in nature. In several cases of nonlinear systems, a small change in a parameter can lead to sudden and dramatic changes in both the qualitative and quantitative behaviour of the system. Recently, chaos and fractals are most popular topics of exploration by researchers. The main purpose of this presentation is to demonstrate chaos and fractals in nonlinear dynamical systems.

Extended convergence analysis of Noor-Waseem iterative method

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Noor and Waseem in 2019 introduced a new two-step iterative method of Newton's variation for solving the nonlinear equation $F(x) = 0$. It was proved using the conventional Taylors expansion, that the order of convergence of this method is three, better than that of the Newton's method. But, their proof forces the operator involved to be four times differentiable. This is a major drawback, since the method will not be applicable to those functions that are not four times differentiable. Further, no computable error bounds are provided and the local convergence analysis is provided only when the space is $\mathbb{R}^m$. The more interesting semi-local convergence analysis is also not provided.

In our work, all the above mentioned limitations are addressed effectively in the more general Banach space setting. The convergence order is proved to be three, without the application of the Taylors expansion. Also, our study does not force the operator involved to be four times differentiable. Instead, imposes conditions only on the first and second derivative, making the method available to solve wider range of practical problems. Further, this work extends the method to two more methods yielding better orders, five and six. The local convergence analysis of all the three methods are discussed. Moreover, the semi-local convergence analysis that gives information about the initial point to be chosen, is discussed in detail for all the methods discussed. To support the theory, numerical examples and dynamics are provided.

A Review on Mathematical Models of Drug Diffusion in the Human Body

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In this work, a mathematical model has been studied to understand the processes between the human body and the chemical substance "drug." The diffusion process has been used to build three models applying Fick's principle and the law of mass action. In this study, the classical model is fractionalized with Caputo fractional derivatives, and the solutions are derived using Sumudu transform methods to determine the specified model's answer more precisely. Mathematical modeling of drug diffusion is a proper predictive method for understanding the transport process of drugs. Mathematical modeling is theoretical, but the results are set up to give realistic results when empirically confirmed. Local operations in each compartment efficiently execute multiple mathematical models and numerical simulations without experimentation. Numerical parameters have been used to calculate the amount of drug in each compartment. A graph is used to demonstrate how the fractional parameter affects drug concentration. Graphs are made with the help of MAPLE software.

On (ϕ, φ)-best proximity points for proximal contraction mappings

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The aim of this contribution is to define two classes of proximal contractions by means of functions endowed with monotone type properties, continuity and the existence of a constant that verifies an equality involving limits. The existence of (ϕ, φ)-best proximity points are stated and proved with respect to mapping which fulfill axioms defined by the use of the proximal contractions. The best proximity points are also a part of the set of zeros for the lower semi continuous functions ϕ, φ. Examples and consequences of these results are also provided.

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Fractals as Mandelbrot and Julia sets of Exponential Function through Krasnoselskij Iteration

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In our present study, we explore new variants of Julia and Mandelbrot sets, employing the Krasnoselskij Iteration method for fixed-points. We establish escape criteria for exponential functions and subsequently employ this technique to reveal novel variants of these sets. We observe that the elapsed time generally surpasses the average number of iterations (ANI) when comparing computational times for these variations while increasing the value of 'n' intensifies color and enlarges fractals.

Keywords: fractals, fixed-point iteration, mandelbrot sets, julia sets, exponential function, krasnoselskij iteration

Results concerning to exact controllability of Sobolev-type Hilfer fractional differential system using measure of noncompactness

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This paper is concerned with the existence and exact controllability results for the neutral Sobolev- type differential system using Hilfer fractional derivative with nonlocal initial conditions in a Banach space. To establish sufficient conditions for the proposed control problem, the theory of measure of noncompactness, propagation family of linear operators combined with Sadovskii's fixed point approach is used. Finally, we provide an example to illustrate our main results.

Keywords: Exact controllability, Neutral, Sobolev-type, Hilfer fractional derivative, Sadovskii's fixed point theorem, Measure of non-compactness.

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Interpolative contractive results of fixed point over quasi-partial b-metric space

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In the early years of the 20th century, the renowned mathematician Banach[1] commenced the concept of the Banach Contraction Principle. Due to its consequences and feasible implementations, the idea has been enlarged and generalized in various directions [2-6]. Recently, Karapinar [7] adopted an interpolative approach to establish fixed point results in the setting of complete metric space. In 2016, Gupta and Gautam [8] defined a new metric space known as quasi- partial b-metric space and proved fixed point theorems on this space. Inspired by these results, we have defined the new contractive results in the framework of quasi-partial b-metric space and proved the corresponding fixed point theorem by adopting the notion of interpolation. The results are further validated with the application based on them. The applications of interpolative contraction in sensitivity analysis of experimental signals and synthesis of scientific data where approximation of natural curves and surfaces is needed, are illustrated here. Examples are given which are based on the new approach.

Keywords: Chatterjea type contraction, Kannan Contraction, quasi-partial b-metric space, interpolative contraction, fixed point

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Asymptotic behavior for a viscoelastic wave equation with past history

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A coupled system of nonlinear viscoelastic wave equation with Balakrishnan-Taylor damping, infinite memory and distributed delay terms is studed. By considered the kernels $h_i : \mathbb{R}_+ \to \mathbb{R}_+$ satisfying where ε_i and H_i are functions satisfying some specific properties, and under this very general hypothesis on the behavior of h_i at infinity and by drop the boundedness hypothesis in the history data, we show the stability of the system.

Connection between the packing problem of disks on torus and the effective conductivity of composites

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The work is devoted to recently established connection between the packing problem of disks on torus and the effective conductivity of composites with circular inclusions. The packing problem is usually investigated by geometrical arguments, the conductivity problem by means of elliptic functions. An algorithm is developed in order to determine the optimal location of two disks on torus formed by the hexagonal lattice and square lattice. The corresponding minimization function is constructed in terms of expressions consisting of elliptic functions with unknown arguments. The numerically found roots coincide with the previously established optimal points by a pure geometrical study.

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